Systematic Errors and NMR Composite pulses

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Setting the scene: Hamiltonians

$$\mathrm{i}\frac{\partial}{\partial t}|\psi\rangle = \mathcal{H}|\psi\rangle$$

$$\mathcal{H} = \mathcal{H}_i + \mathcal{H}_e(t)$$

System Hamiltonian

Control Fields

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle$$

$$U(t) = \mathcal{T}\exp\left(-\mathrm{i}\int_0^t H(t')\mathrm{d}t'\right)$$

$$U(t) = \prod_n U_n$$

$$U_n = \exp\left(-\mathrm{i}\mathcal{H}_n\tau_n\right)$$

Errors

- We don't know $\mathcal{H}_\text{i}$ exactly
- We can't control $\mathcal{H}_\text{e}$ exactly

- So total evolution *U* will be wrong

- Is there some way of making *U* robust to small errors in $\mathcal{H}_\text{i}$ and $\mathcal{H}_\text{e}$?

Systematic Errors

- Random errors are completely unknown
  - Have to use error correction or DFS approaches
- Systematic errors are unknown but constant
- We can use reproducibility to reduce errors!

- In chaotic systems small errors build up catastrophically; we need anti-chaos where errors systematically cancel each other out!

Calibration

- If errors are constant why not just characterise them and calibrate them out?
- Works well for some errors, but consider
  - Multiple qubits interacting with same fields
  - Macroscopic ensembles in NMR systems
  - Temporal ensembles and slowly varying fields
- RF amplifier power often oscillates with a period of about 20 minutes (temperature)

Systematic Errors

- Almost any QIP system with control fields will suffer from time variations in these fields
- Many systems will also have some sort of spatial ensemble

- Our aim is to produce logic gates which are robust to a distribution of control fields

NMR

- The methods I will describe were mostly developed for use in NMR and many papers use NMR language
- Important to know the basics! But don't worry too much about the details

NMR spin Hamiltonian

$$\mathcal{H}_i = \omega_1 \frac{\sigma_z^1}{2} + \omega_2 \frac{\sigma_z^2}{2} + \omega_{12} \frac{\sigma_1 \cdot \sigma_2}{4}$$

Larmor frequency

Zeeman term

Spin–spin coupling

- Assumes a system of two spin-1/2 nuclei in the liquid state
- Hamiltonian is much more complex in the solid state
- Spin-0 nuclei can obviously be ignored
- High-spin nuclei can be largely ignored (complex reasons)

Weak coupling

$$\mathcal{H}_i = \omega_1 \frac{\sigma_z^1}{2} + \omega_2 \frac{\sigma_z^2}{2} + \omega_{12} \frac{\sigma_z^1 \sigma_z^2}{4}$$

Larmor frequency

Zeeman term

Spin–spin coupling

$$|\omega_{12}| \ll |\omega_1 - \omega_2|$$

- Weak coupling approximation is to keep only the diagonal terms. The coupling is *truncated* by the Zeeman terms (equivalent to first order perturbation theory)
- Good approximation in most NMR QIP systems

Product Operators

$$\mathcal{H}_i = 2\pi\nu_I I_z + 2\pi\nu_S S_z + \pi J\, 2I_z S_z$$

- Traditional notation developed in NMR
- Spins called *I* and *S* and spin–spin coupling called *J*
- Factors of ½ are incorporated into the spin operators
- Factors of 2 get moved around in ways that look strange unless you are used to it (and even then are a bit odd)

- Fundamentally equivalent to ordinary notation
- Don't worry too much about it

Multi-spin Hamiltonian

$$\mathcal{H}_i = \tfrac{1}{2}\sum_j \omega_j \sigma_z^j + \tfrac{1}{4}\sum_{j<k} \omega_{jk} \sigma_z^j \sigma_z^k$$

- Weak coupling approximation assumed
- Some spin–spin couplings may be negligible

Remember that this Hamiltonian only applies for spin-1/2 nuclei in the liquid state!

Energy scales

- The NMR transition frequency depends on the *gyromagnetic ratio* of the nucleus and the *magnetic field strength*
- Typical NMR transitions occur in the frequency range 50MHz to 800 MHz
- Very low energy compared with *kT* at room temperature!

Ensembles

- The NMR transition frequency is far too low to detect single photons directly
- Instead use macroscopic ensembles with many identical copies of each qubit

- Can’t achieve spatial localisation!
- Can’t achieve projective measurements!
- Spin states are (almost) always highly mixed!

Pseudo pure states

$$\rho_{pp} = (1-p)\frac{\mathbb{1}}{2^n} + p|\psi\rangle\langle\psi| \qquad\qquad p \sim \frac{10^{-4}}{2^n}$$

- Can be prepared in various ways not described here
- The maximally mixed state does not evolve and cannot be detected by NMR measurements so behaviour is *identical* to that of corresponding pure states

- The effective purity if the state *p* is very low and scales exponentially badly with *n*, the number of qubits, making large scale NMR QIP impractical

Coherent control

- The low transition frequency means that are *always* in the *coherent control* regime

- Spontaneous lifetimes are *extremely* long
- Stimulated decay lifetimes are very long

- Easy to make strong coherent RF sources

Pulses

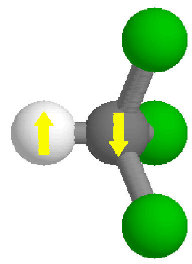
- A pulse is a short period of applied RF near resonance with one or more spins
- Spins near resonance are strongly affected
- Best understood in the *rotating frame* and making the *rotating wave approximation*

- Spins far from resonance evolve under the background Hamiltonian

Heteronuclear & Homonuclear

- In heteronuclear systems all the spins are of different nuclear species and so have very different Larmor frequencies
- In homonuclear systems two or more spins are of the same nuclear species and so have very similar Larmor frequencies, differing only by small *chemical shifts*
- Heteronuclear systems are much simpler!

Heteronuclear systems



- Control fields only affect one spin at a time
- Work in a multiply rotating frame to remove all the Larmor frequencies

$$\mathcal{H} = \mathcal{H}_0 + \sum_j \mathcal{H}_j(t) \qquad \mathcal{H}_0 = \tfrac{1}{4}\sum_{j<k} \omega_{jk}\sigma_z^j\sigma_z^k$$

$$\mathcal{H}_j = \tfrac{1}{2}\Omega_j(t)\left(\sigma_x^j \cos\left[\phi_j(t)\right] + \sigma_y^j \sin\left[\phi_j(t)\right]\right)$$

Pulse amplitude

Pulse phase

Pulses and delays

- Both conventional NMR experiments and NMR QIP experiments are implemented by alternating short pulses and long delays
- As long as pulses are short enough can neglect evolution under $\mathcal{H}_0$
- Delays are used to implement two-qubit gates through spin–spin couplings

Single qubit gates

- On resonance pulses implement rotations
- Write as θ_ϕ where $\theta=\Omega\tau$ is the angle of rotation and ϕ is the azimuth (phase) angle of the rotation in the *xy*-plane
- Can describe angles in radians or degrees or (for phase angles) using axis letter codes
- Other gates can be implemented as networks of pulses

NOT gate

- The NOT gate is a 180° rotation around *x*

$$\text{NOT} = 180_x = \pi_x = 180_0 = \pi_0$$

- Only right up to (irrelevant) global phase
- Note that we consider this as a 180° evolution under $\sigma_x/2$ and not a 90° evolution under σ_x
- Other conventions are used leading to enormous potential for confusion!

Hadamard gate

- The Hadamard gate is a 180° rotation around a tilted axis and is best implemented using a sequence of pulses

$$\mathrm{H} = 90_y 180_x = 180_x 90_{-y}$$

- Note that pulses are applied from left to right, the opposite way from propagators!

Phase gates

- Phase gates are equivalent to z-rotations and can be implemented using various sequences

$$\theta_z = 90_y \theta_x 90_{-y}$$

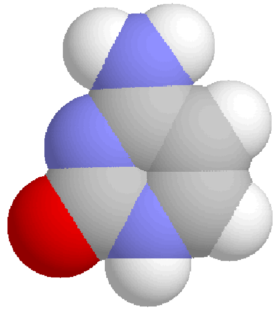
$$\theta_z = 180_\phi 180_{\phi+\theta/2}$$

- First approach (composite *z*-rotation) more common in conventional NMR, second approach more common in NMR QIP

Two qubit gates

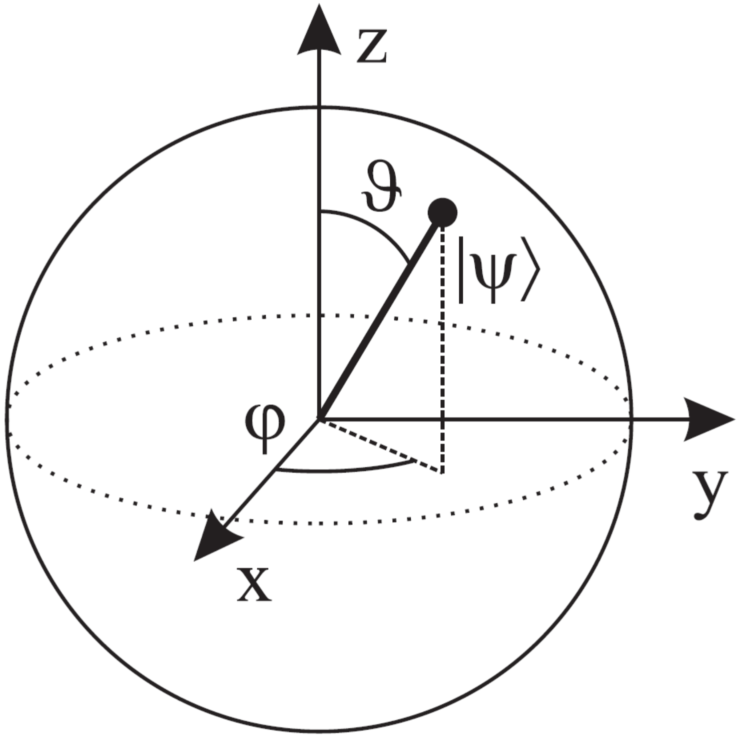
- Delays are used to implement two-qubit gates through spin–spin couplings
- Background Hamiltonian contains a complex network of couplings
- Can be simplified using *spin echoes* to remove unwanted couplings. See any standard text!

Homonuclear systems



- A control field can in general affect two or more spins
- Low amplitude control fields restore selection but pulses now long compared with couplings
- Gets complicated!
- So ignore it for the moment…

Bloch Sphere



States of a qubit can be described in spherical polar coordinates and then mapped onto points on the surface of the Bloch sphere

Mixed states live *inside* the sphere

$$|\psi\rangle = \cos(\vartheta/2)|0\rangle + \sin(\vartheta/2)e^{i\varphi}|1\rangle$$

Bloch vector

- Bloch vector points from origin of the Bloch sphere to the qubit state
- Can also derive it from density matrices

$$|\psi\rangle\langle\psi| = \tfrac{1}{2}\left(\mathbb{1} + s_x\sigma_x + s_y\sigma_y + s_z\sigma_z\right)$$

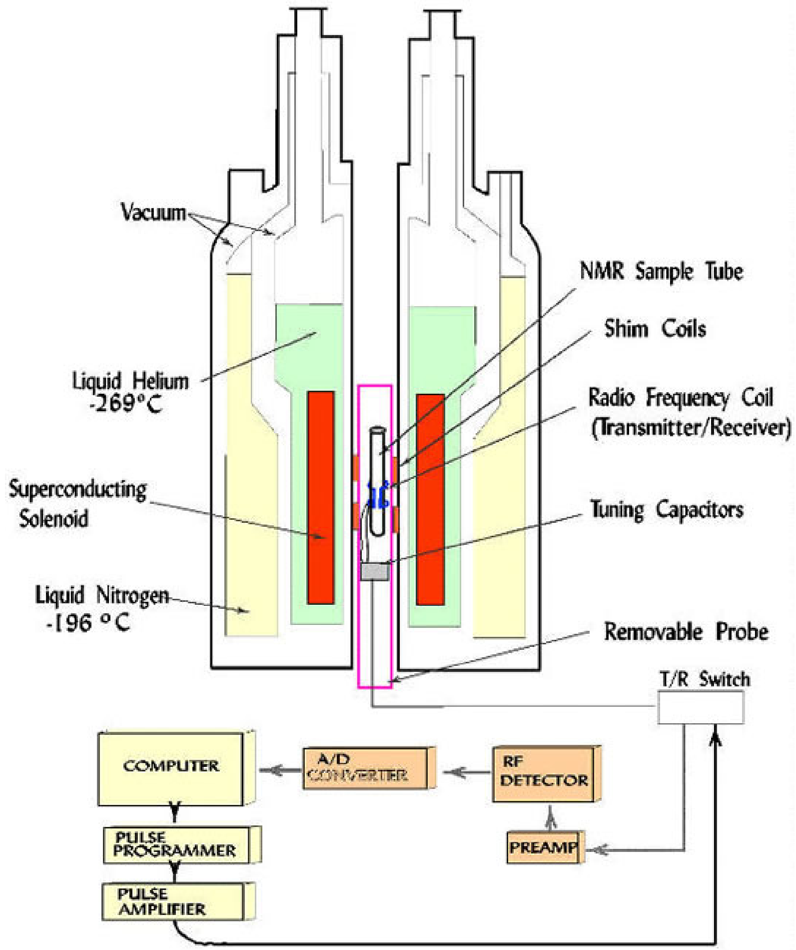
$$s_x = \sin\vartheta\cos\varphi \quad s_y = \sin\vartheta\sin\varphi \quad s_z = \cos\vartheta$$

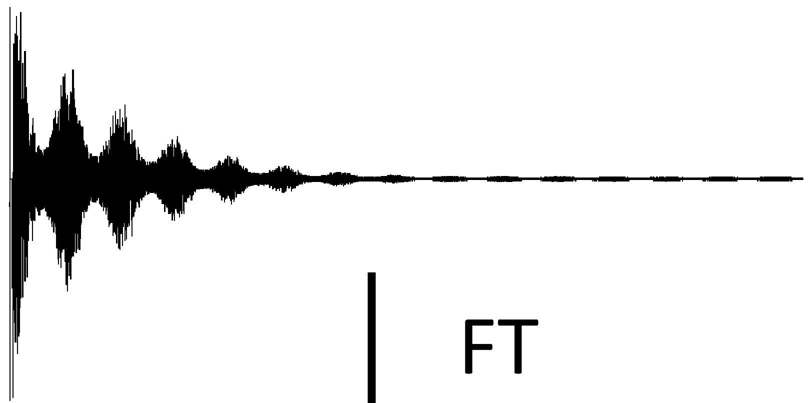
- ***s*** is a unit vector for a pure state

Conventional NMR

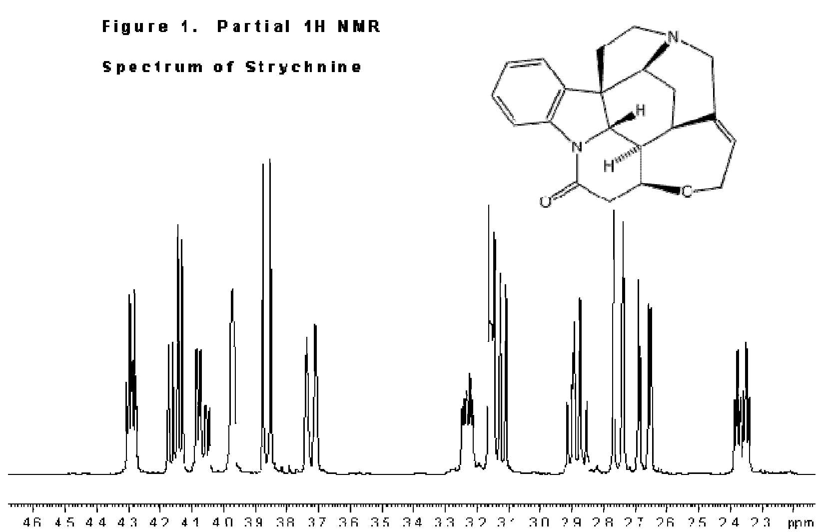
- Spin starts off in thermal state along +z axis
- 90° pulse rotates spin into xy-plane where it precesses at the Larmor frequency
- Observe magnetisation and Fourier transform to get spectrum of transitions
- Usually described using Bloch vectors with some unusual sign conventions

Conventional NMR

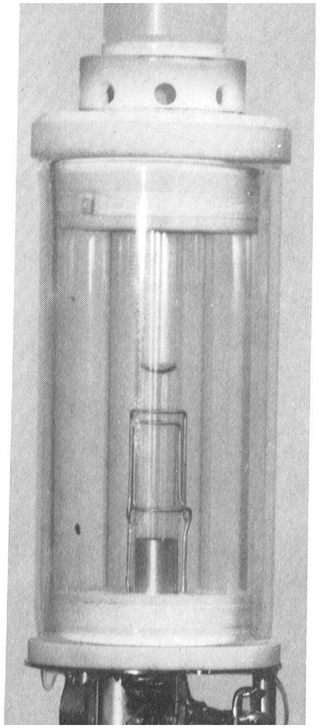




FT



Pulse length errors

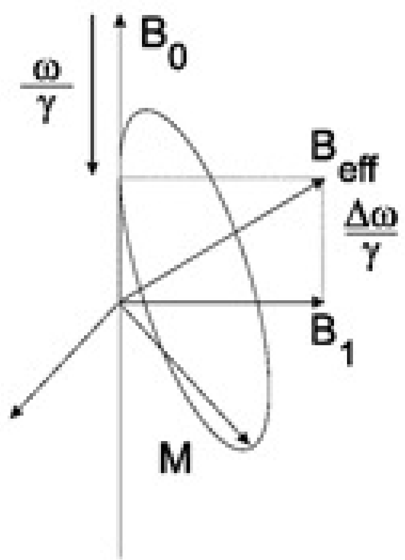


- Really *pulse strength errors*

- The RF coil produces a magnetic field which varies over the sample
- Control fields vary by ±10% over most of sample and more at edges
- Rotation angles vary in proportion

- Coils optimised for field strength not homogeneity

Off-resonance effects



- Occur when the control field is not exactly on resonance with a transition frequency
- Hamiltonian is the sum of the RF field and an off resonance term
- Evolution occurs around tilted axis

- Important in *homonuclear* spin systems

Errors

$$U(\theta,\phi) = \exp\left[-\mathrm{i}\theta\left(\sigma_x \cos\phi + \sigma_y \sin\phi\right)/2\right]$$

$$U(\theta,0) = \cos(\theta/2)\mathbb{1} - \mathrm{i}\sin(\theta/2)\sigma_x$$

$$\begin{aligned} V(\theta,0) &= U\left(\theta(1+\varepsilon),0\right) \\ &= U(\theta,0) - \varepsilon \times \tfrac{1}{2}\theta\left[\sin(\theta/2)\mathbb{1} + \mathrm{i}\cos(\theta/2)\sigma_x\right] \\ &+ O\left(\varepsilon^2\right) \end{aligned}$$

Pulse length error is first order in ε

Fidelities

$$\mathcal{F}(U,V) = \left| \frac{\mathrm{tr}\left(U^{\dagger}V\right)}{2^{n}} \right|$$

Hilbert–Schmidt inner product of propagators

$$= \left|\cos\left(\varepsilon\theta/2\right)\right|$$

$$= 1 - \varepsilon^{2}\theta^{2}/8 + O\left(\varepsilon^{4}\right)$$

Sometimes defined as the square of this formula instead; the difference is not very important but watch out!

Infidelities

$$\mathcal{I}(U,V) = 1 - \mathcal{F}(U,V)$$
$$= \varepsilon^2\theta^2/8 + O\left(\varepsilon^4\right)$$

Simple pulses have second order infidelity in e

Note that n^{th} order errors give $2n^{\text{th}}$ order infidelity

Using the square of the fidelity definition just doubles the infidelity for small errors

Point-to-point fidelity

$$\mathcal{P}\left(U,V,\left|\psi\right\rangle\right)=\left|\left\langle\psi\right|U^{\dagger}V\left|\psi\right\rangle\right|^{2}$$

Sometimes use the square root of this formula

Common in conventional NMR where particular initial states are very important

Can extend to QIP by averaging over all initial states

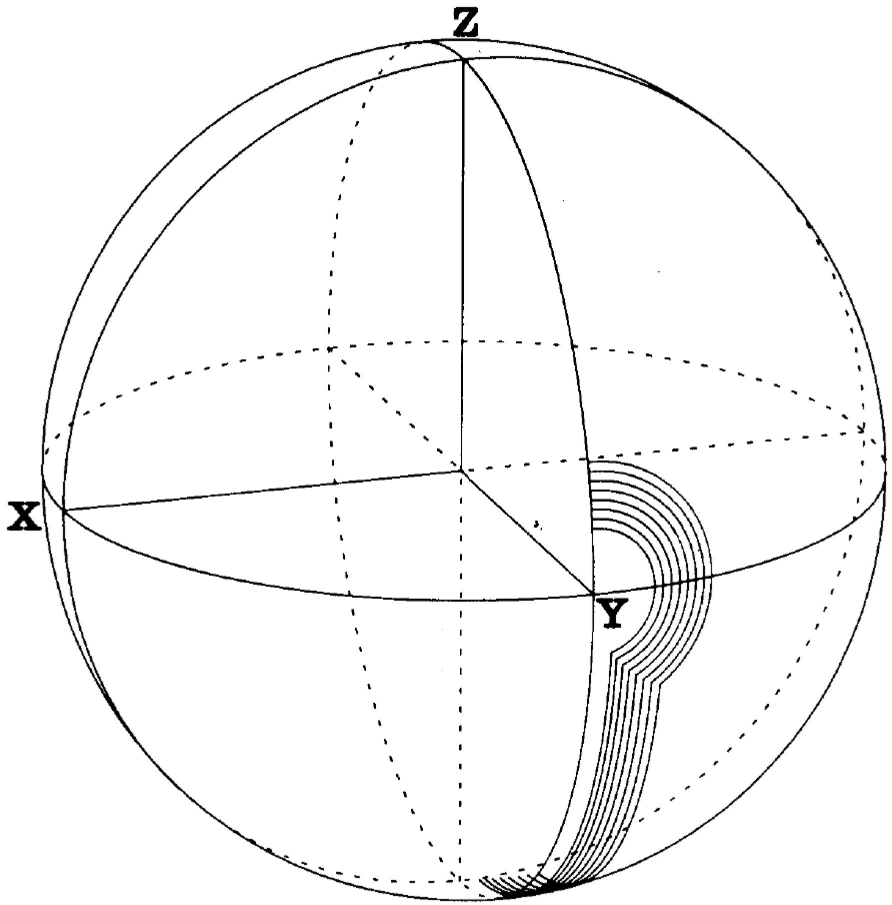
Composite pulses

- Widely used in conventional NMR to tackle pulse length errors and off-resonance effects

- Replace a single pulse by a sequence of pulses with same overall effect but greater tolerance of errors

Invented by Malcolm Levitt (Oxford Chemistry) during his undergraduate project!

Composite inversion



$90_x 180_y 90_x$

Designed for point-to-point transfer from +z to –z in presence of pulse length errors.

Easily seen on Bloch sphere: error in outer pulses is largely corrected by inner pulse

Composite inversion

- Point to point fidelity is greatly improved

$$\mathcal{P}_{naive} = 1 - \varepsilon^2 \pi^2 / 4 + O\left(\varepsilon^4\right)$$

$$\mathcal{P}_{Levitt} = 1 - \varepsilon^4 \pi^4 / 16 + O\left(\varepsilon^6\right)$$

- But overall fidelity is unchanged!

$$\mathcal{F} = 1 - \varepsilon^2 \pi^2 / 4 + O\left(\varepsilon^4\right)$$

On average Levitt's composite pulse is no better than a simple pulse (does better for some initial states but worse for others)

Fully compensating pulses

- For QIP we need a pulse that is error tolerant for *any* initial state
- Rarely needed in conventional NMR, but a few were designed as curiosities

- Known as *fully compensating pulses*, or *Class A composite pulses* or *general rotors*

Tycko's pulse

Tycko's composite inversion pulse $180_{60}180_{300}180_{60}$ will perform 180_x with compensation of pulse length errors for any initial state

$$\mathcal{F}_{Tycko} = 1 - \varepsilon^4 \times 3\pi^4 / 128 + O\left(\varepsilon^6\right)$$

A robust NOT gate!

SCROFULOUS pulses

$$\text{replace } \theta_x \text{ with } \beta_{\phi_1} 180_{\phi_2} \beta_{\phi_1}$$

$$\beta = \operatorname{arcsinc}\left(\frac{2\cos(\theta/2)}{\pi}\right) \quad \operatorname{sinc}(x) = \sin(x)/x$$

$$\phi_1 = \arccos\left(\frac{-\pi\cos\beta}{2\beta\sin(\theta/2)}\right) \quad \phi_2 = \phi_1 - \arccos\left(\frac{-\pi}{2\beta}\right)$$

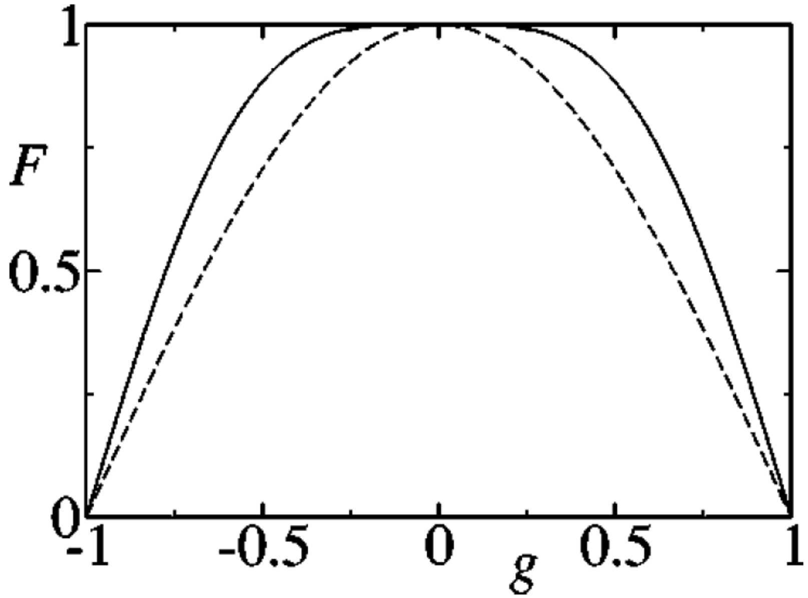
In the case θ=180° Tycko's result is recovered; otherwise have to solve numerically

SCROFULOUS pulses

θ	β	ϕ_1	ϕ_2
30	93.0	78.6	273.3
45	96.7	73.4	274.9
90	115.2	62.0	280.6
180	180	60	300

Pulse angles for some choices of θ_x

Change pulse phase by offsetting ϕ_1 and ϕ_2



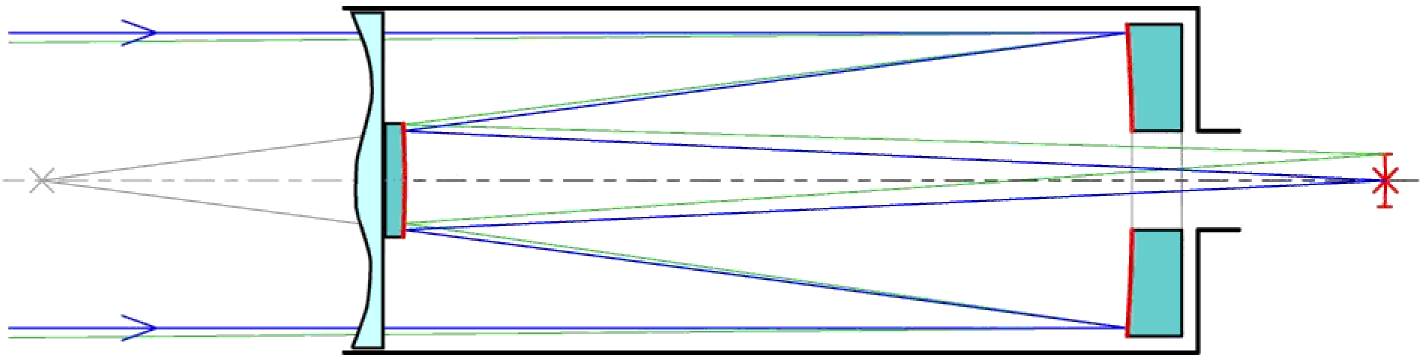
Fidelity as a function of pulse length error for 180° pulse (NOT gate)

Designing composite pulses

- Tycko’s used a Magnus expansion to get a series expansion of the propagator in the error and set the first order error term to zero
- Have to also make the error free propagator do the right thing
- Finding a *simultaneous solution* to these two targets is difficult in the general case
- Depends on lucky initial guesses

Wimperis and error correction

- Wimperis's key idea was to separate the two parts of the problem by combining an error prone main pulse with an *error correcting* "do-nothing" pulse



BB1

$$180_{\phi_1}180_{\phi_2}180_{\phi_3}180_{\phi_4} = \left[-2\left(\phi_1 - \phi_2 + \phi_3 - \phi_4\right)\right]_z$$

$$\text{choose } \phi_1 = \phi_4 \text{ and } \phi_2 = \phi_3$$

$$180_{\phi_1}360_{\phi_2}180_{\phi_1} = \mathbb{1}$$

In the absence of errors this sequence does nothing.
What happens with pulse length errors?

BB1 with errors

$$W\left(\phi_1,\phi_2\right)=V\left(\pi,\phi_1\right)V\left(2\pi,\phi_2\right)V\left(\pi,\phi_1\right)$$
$$=1-\varepsilon\times\mathrm{i}\pi\left(C\sigma_x+S\sigma_y\right)+O\left(\varepsilon^2\right)$$

$$C=\cos\left(\phi_1\right)+\cos\left(2\phi_1-\phi_2\right)$$
$$S=\sin\left(\phi_1\right)+\sin\left(2\phi_1-\phi_2\right)$$

choosing $\phi_2=3\phi_1$ sets $S=0$

BB1 with errors

$$W\left(\phi_1, 3\phi_1\right) = 1 - \varepsilon \times \mathrm{i}\pi 2\cos\phi_1\sigma_x + O\left(\varepsilon^2\right)$$
$$\approx U\left(\varepsilon \times 4\pi\cos\phi_1, 0\right)$$

This sequence generates a *pure error term*, a rotation around σ_x with an angle proportional to the error ε and depending on ϕ_1.

BB1 with errors

The main pulse is equal to a *perfect* θ_0 pulse followed by a rotation by $\varepsilon\theta$

$$V(\theta,0) = U(\varepsilon\theta,0)\,U(\theta,0)$$

$$W(\phi_1,3\phi_1) \approx U(\varepsilon 4\pi\cos\phi_1,0)$$

Can make the two error terms cancel: choose

$$4\pi\cos\phi_1 = -\theta \quad \Rightarrow \quad \phi_1 = \pm\arccos(-\theta/4\pi)$$

BB1

Wimperis placed the correction sequence *before* the main pulse

$$V(\theta,0)W(\phi_1,3\phi_1)$$

but can put it after the pulse or in the middle

$$V(\theta/2,0)W(\phi_1,3\phi_1)V(\theta/2,0)$$

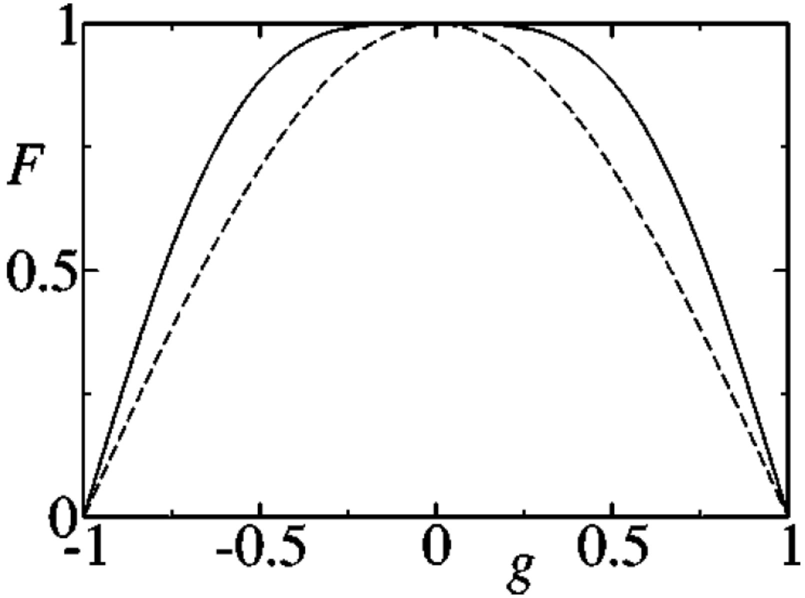
BB1 fidelity

- Removes the 1st order error term so expect the infidelity to be 4th order in ε, but actually

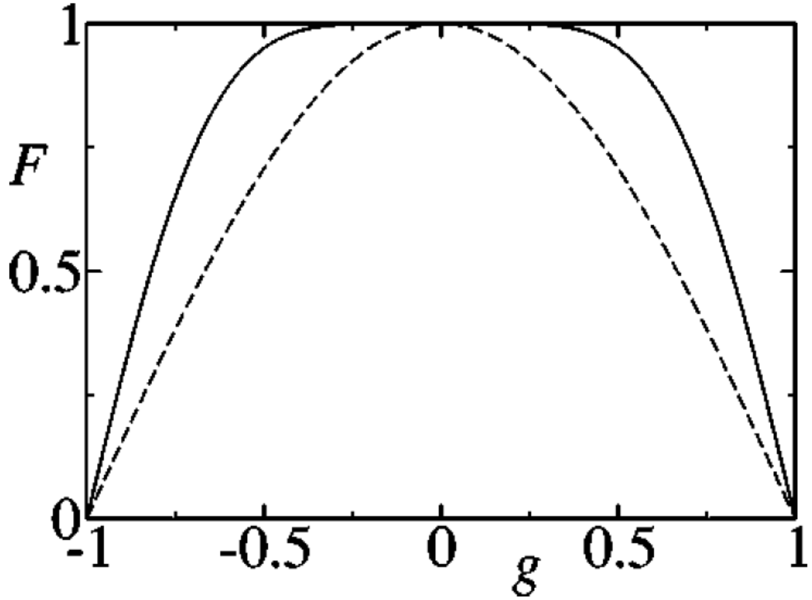
$$\mathcal{F}_{BB1} = 1 - \varepsilon^6 \times 5\pi^6 / 1024 + O\left(\varepsilon^8\right) \quad \text{for } 180^\circ \text{ pulse}$$

- Simultaneous removal of the 2nd order error term! Very nice, but no obvious explanation
- Fidelity depends on θ but independent of where the W pulses are placed

Fidelities and pulse length errors



SCROFULOUS

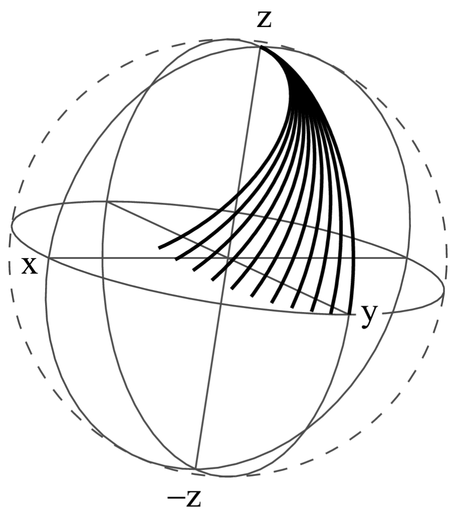


BB1

Fidelities of NOT gates in presence of pulse length errors (dashed line is naïve pulse)

Off-resonance effects

- Consider a control field of strength ω_1 at an offset $\delta\omega$ from resonance. What matters is the *off-resonance fraction* $f=\delta\omega/\omega_1$
- Rotation occurs around a tilted axis



Off-resonance effects on a 90° excitation pulse for off-resonance fractions *f* in the range 0 to 1 in steps of 0.1

Errors and infidelity

$$R(\theta,\phi) = \exp\left[-i\theta\left(\cos\phi\boldsymbol{\sigma}_x + \sin\phi\boldsymbol{\sigma}_y + f\boldsymbol{\sigma}_z\right)/2\right]$$
$$= U(\theta,\phi) - f \times \mathrm{i}\sin(\theta/2)\boldsymbol{\sigma}_z$$

- Error is first order in *f*

$$F_{naive} = \left|\frac{\mathrm{tr}\left(U^{\dagger}R\right)}{2}\right| = 1 - f^2\frac{\sin^2(\theta/2)}{2} + O\left(f^4\right)$$

- Infidelity is second order in *f*

Inverse pulses

$$U(\theta,\phi+\pi)=U(-\theta,\phi) \quad \Rightarrow \quad \theta_{\phi}\theta_{\phi+\pi}=\mathbb{1}$$

Easy to make pulses with negative rotation angles

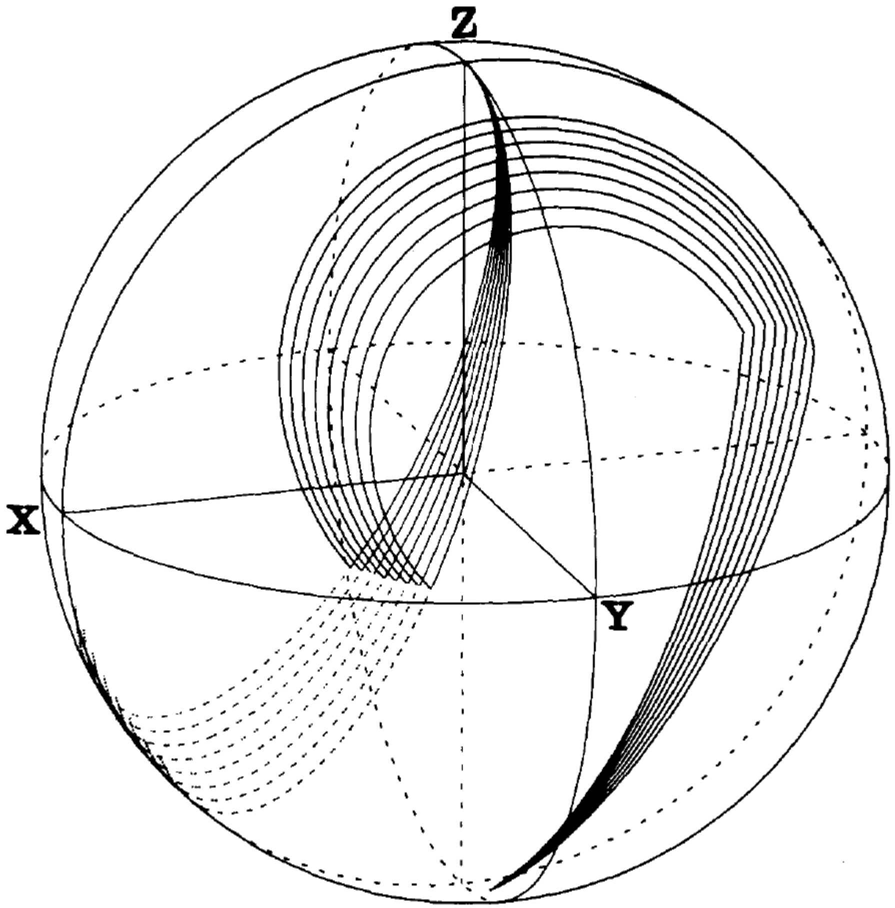
$$V(\theta,\phi+\pi)=V(-\theta,\phi)$$

Still works with pulse length errors

$$R(\theta,\phi+\pi)\neq R(-\theta,\phi)$$

Doesn’t work with off-resonance effects: can only use physical (positive) angles

Composite inversion



$90_x 180_y 90_x$

Designed for point-to-point transfer from +z to –z in presence of off-resonance effects.

Seen on Bloch sphere: error in outer pulses is largely corrected by inner pulse

Tycko's pulse

Tycko also discovered a composite 90° pulse $385_x 320_{-x} 25_x$ which will perform 90_x with moderate compensation of off-resonance effects for any initial state

Discovered by a numerical search but generalised as CORPSE gates

CORPSE pulses

$$\text{replace } \theta_x \text{ by } \alpha_x \beta_{-x} \gamma_x \text{ with } \alpha - \beta + \gamma = \theta$$

$$C(\theta,0) = R(\gamma,0)\,R(\alpha+\gamma-\theta,\pi)\,R(\alpha,0)$$

$$= U(\theta,0) - f \times \mathrm{i}\left(Y\boldsymbol{\sigma}_y + Z\boldsymbol{\sigma}_z\right) + O\left(f^2\right)$$

$$Y = \cos(\gamma - \theta/2) - \cos(\alpha - \theta/2)$$

$$Z = \sin(\gamma - \theta/2) + \sin(\alpha - \theta/2) + \sin(\theta/2)$$

Solve for *Y*=*Z*=0 to remove all first order error terms

CORPSE pulses

$$\text{replace } \theta_x \text{ by } \alpha_x \beta_{-x} \gamma_x \text{ with } \alpha - \beta + \gamma = \theta + 2n\pi$$

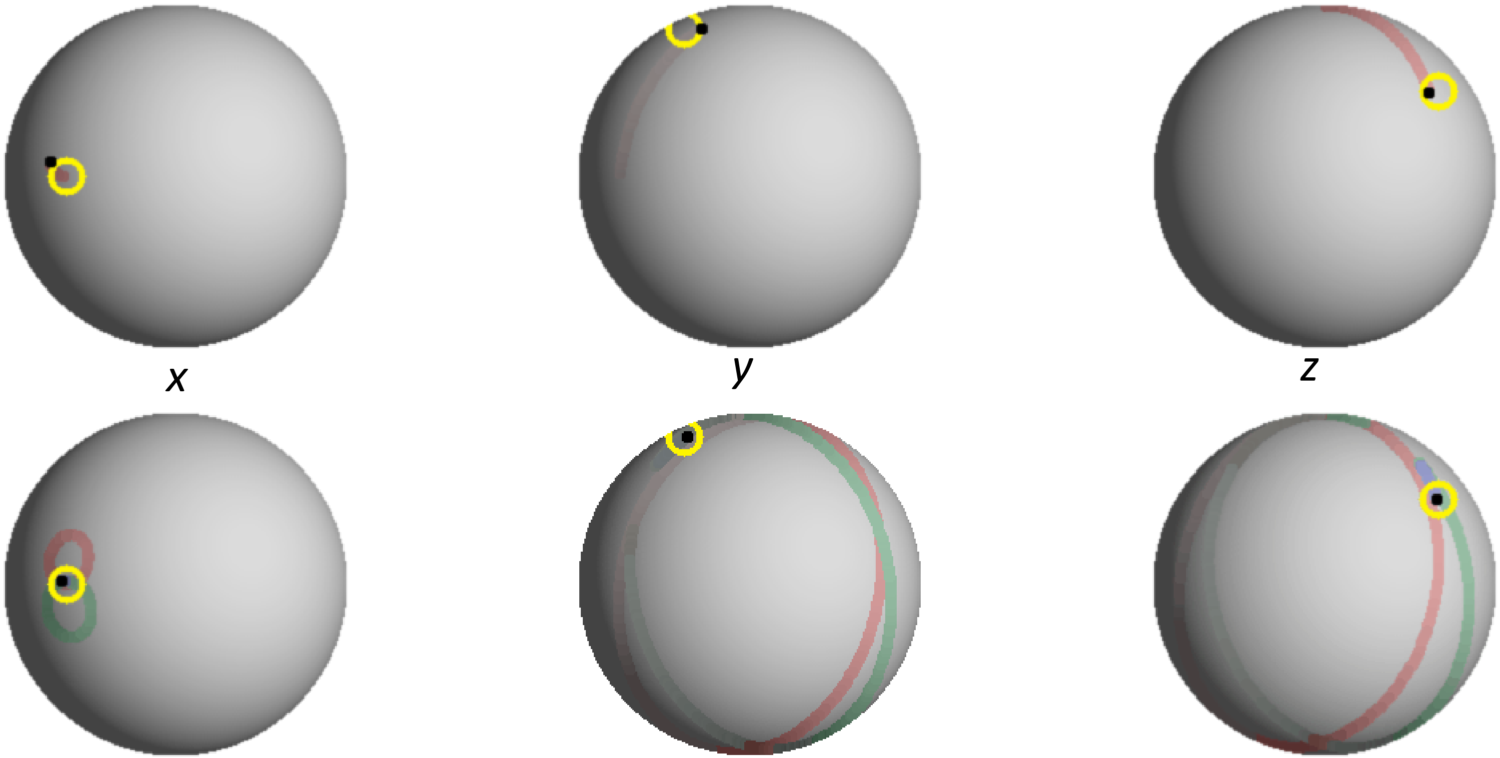
$$\alpha = 2a\pi + \tfrac{1}{2}\theta - \arcsin\left[\tfrac{1}{2}\sin(\theta/2)\right]$$

$$\beta = 2b\pi - 2\arcsin\left[\tfrac{1}{2}\sin(\theta/2)\right]$$

$$\gamma = 2c\pi + \tfrac{1}{2}\theta - \arcsin\left[\tfrac{1}{2}\sin(\theta/2)\right]$$

Choose *a*, *b* and *c* such that all rotation angles are positive. Best results occur at n=0, so a=b=1, c=0.

CORPSE pulses



Trajectories on the Bloch sphere for various initial states after 60° naïve and composite pulses

Infidelity

- Infidelity is fourth order in f with a complex dependence on θ. For case of 180° pulse

$$F_{\text{naive}} = 1 - f^2\left(\frac{1}{2}\right) + O\left(f^4\right)$$

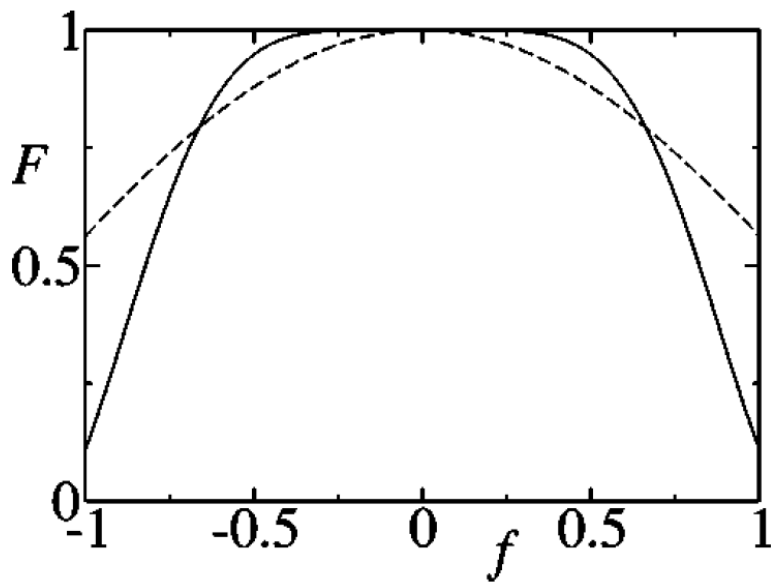
$$F_{\text{CORPSE}} = 1 - f^4\left(\frac{12+\pi^2-4\sqrt{3}\pi}{32}\right) + O\left(f^6\right)$$

CORPSE pulses

θ	α	β	γ
30	367.6	345.1	7.6
45	371.5	337.9	11.5
90	384/3	318.6	24.3
180	420	300	60

Pulse angles for some choices of θ_x

Change pulse phase by offsetting from $\pm x$



Fidelity as a function of off-resonance fraction for 180° pulse (NOT gate)

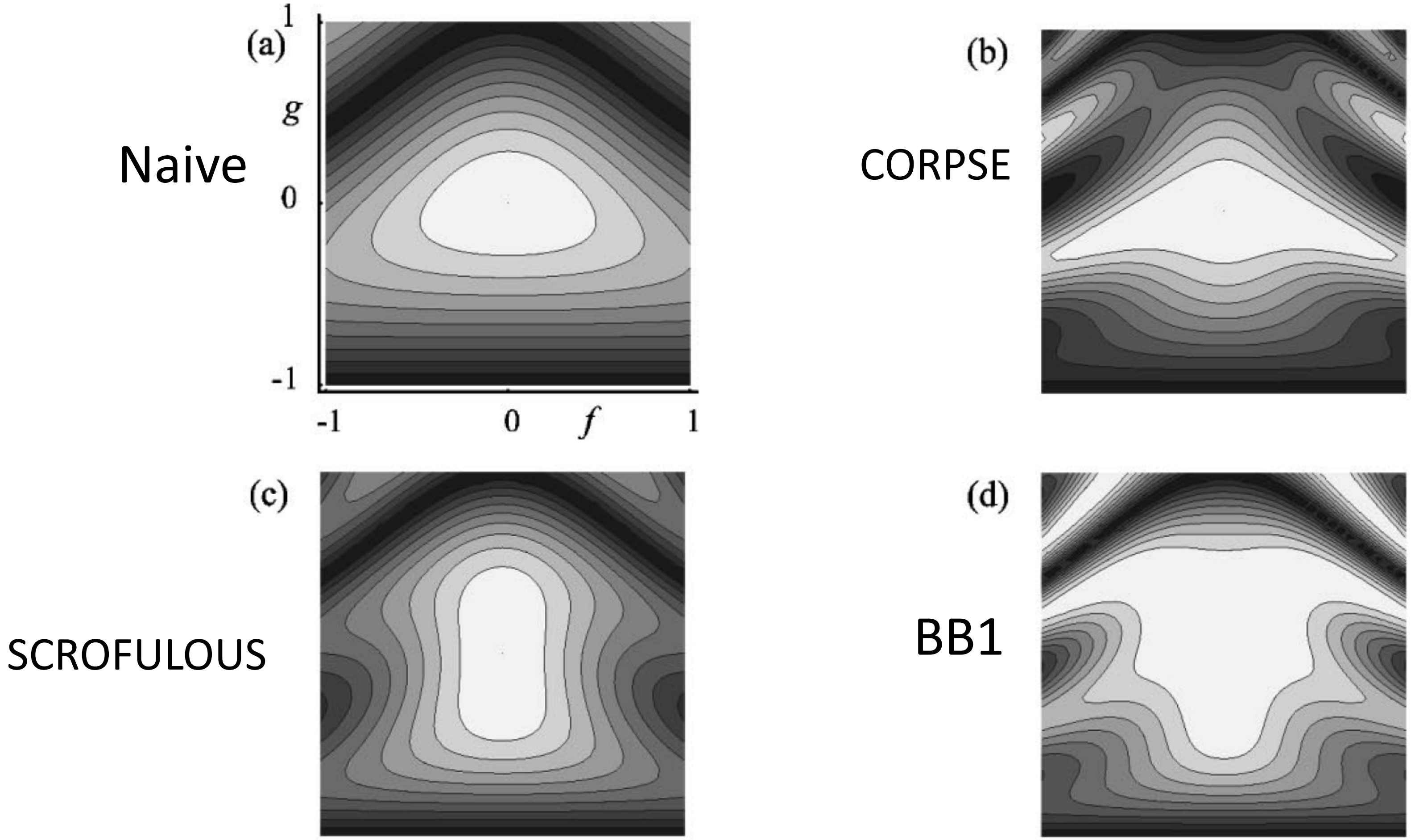
Simultaneous errors

- In the general case *both* pulse length errors and off-resonance effects will occur at the same time!

$$G(\theta,\phi)=\exp\left(-\frac{\mathrm{i}\theta}{2}\left[(1+\varepsilon)\left(\cos\phi\,\sigma_x+\sin\phi\,\sigma_y\right)+f\,\sigma_z\right]\right)$$

- Analysis gets a bit complicated…
- Can explore numerically

Simultaneous errors



Contours plotted at 5% intervals for 180° pulses

Simultaneous errors

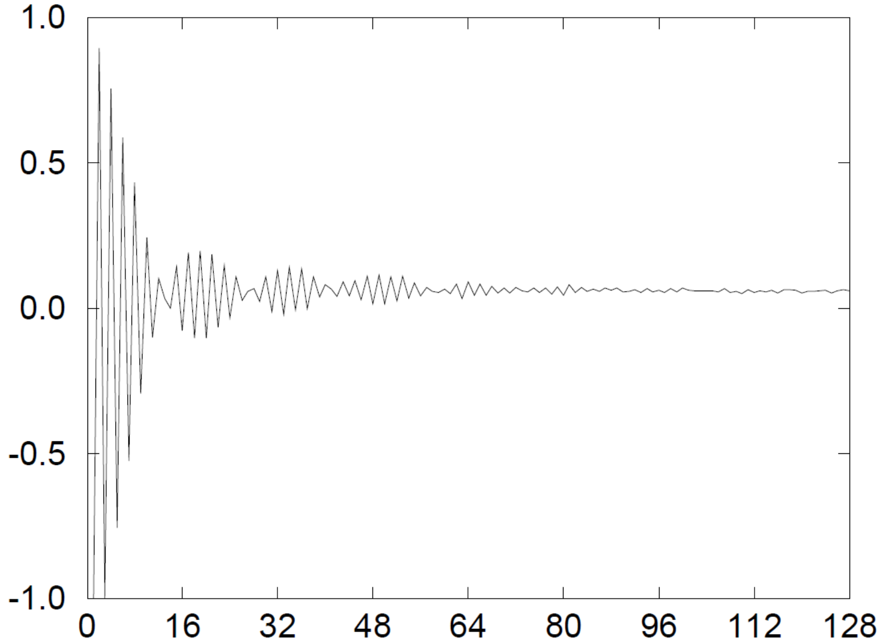
- In CORPSE sequences with no off-resonance effects pulse length errors largely cancel, and looks just like a naïve pulse
- In BB1 sequences with no pulse length errors the off-resonance effects in the W sequence vanish to first order, so it looks almost like a naïve pulse
- SCROFULOUS is more sensitive to off-resonance

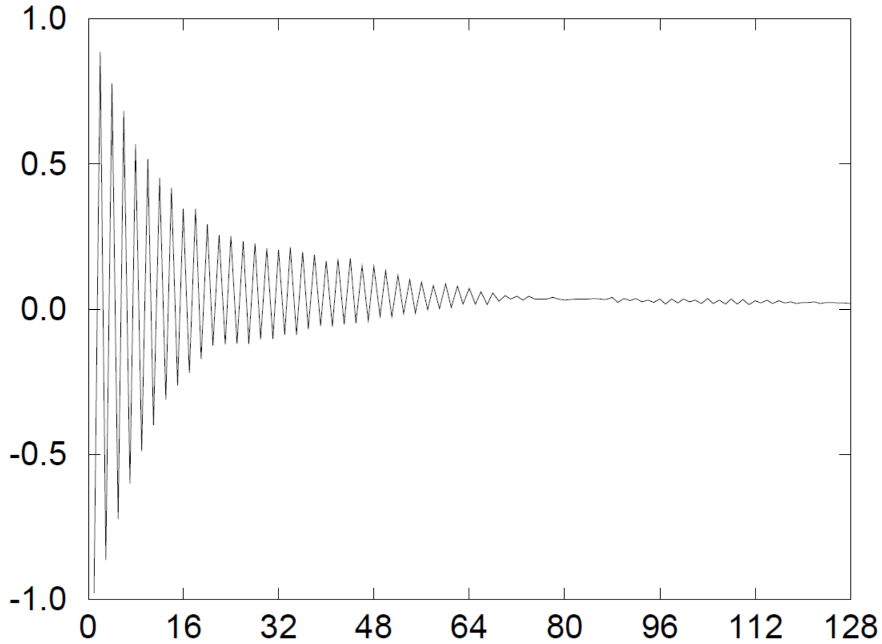
Theory and Practice

- Composite pulses suppress certain systematic errors but are these really dominant?
  - Random errors
  - Phase errors
  - Pulse transients

- *All the proof of a pudding is in the eating*: experiments are the only true test of reality

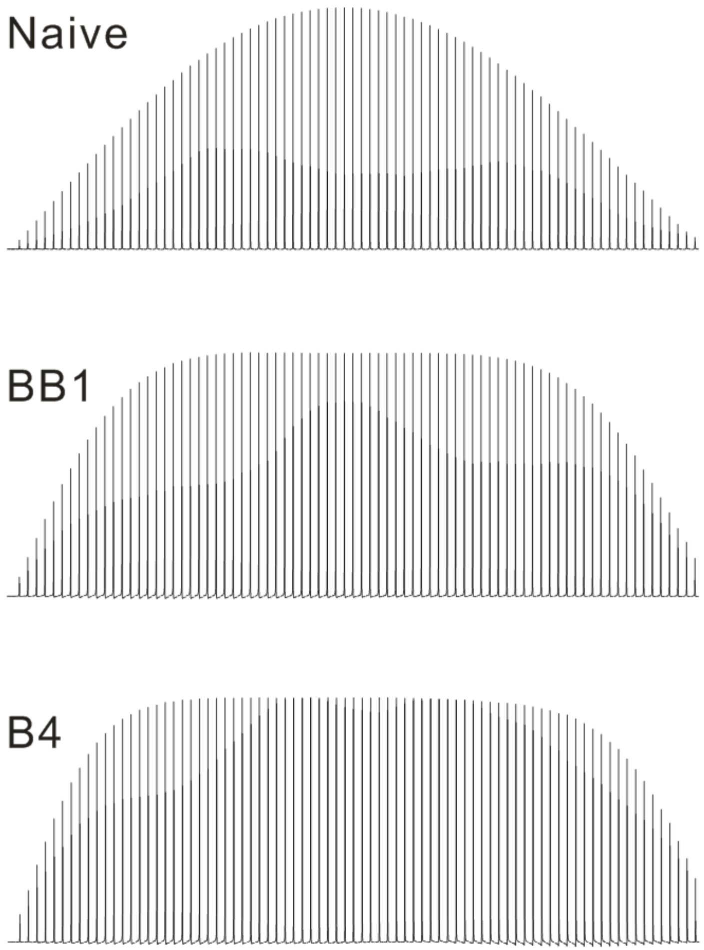
Quantum counting





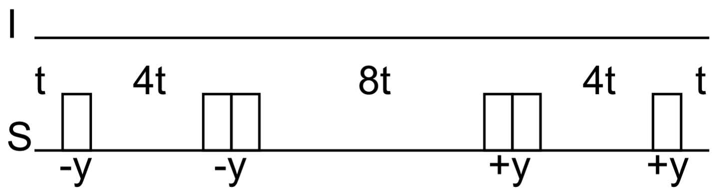
Homonuclear NMR quantum counting experiment. Using CORPSE 90° pulses removes artefacts arising from off-resonance effects.

Pulse length errors

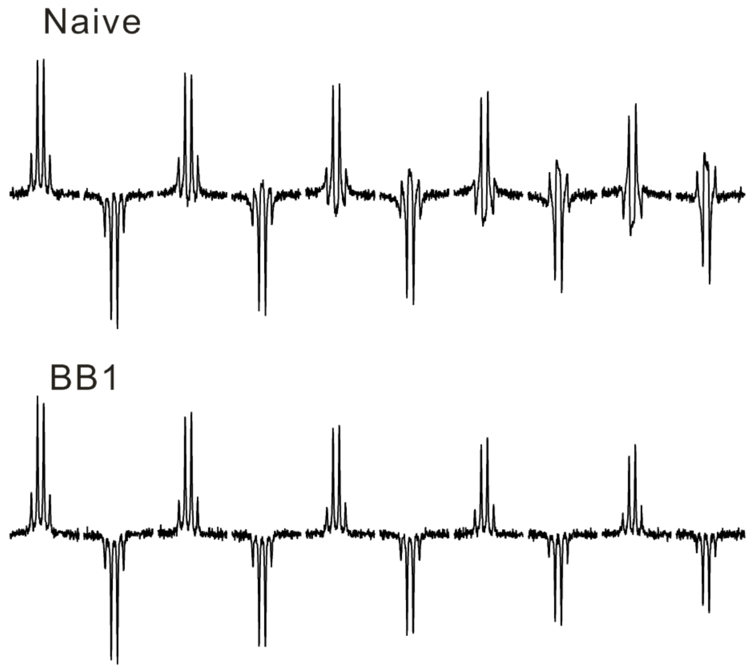


- NMR experiments on a single spin with pulse strength varied by ±100%
- BB1 works very much as expected
- B4 is theoretically better than BB1 but actually performs slightly worse

Two qubit gates

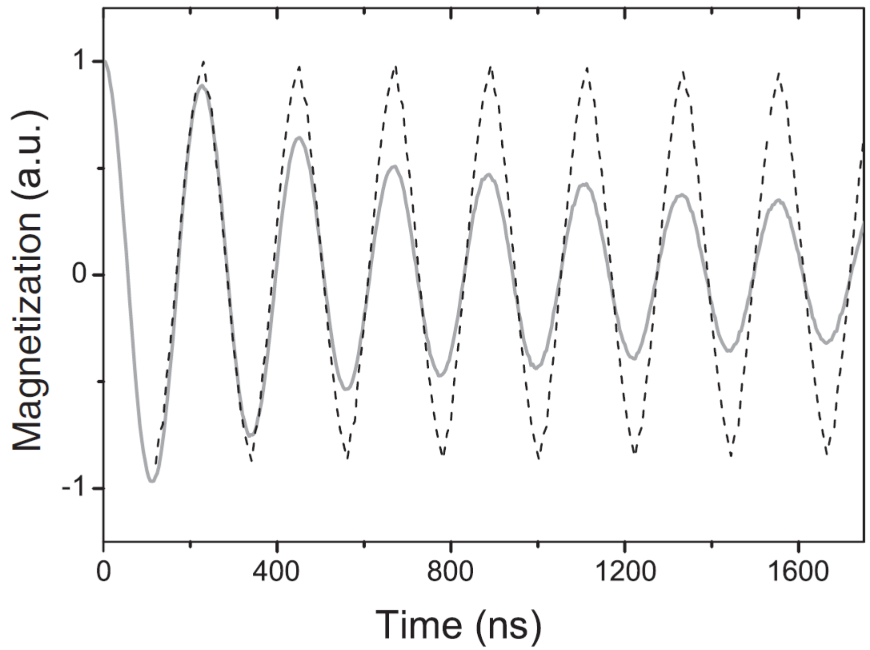


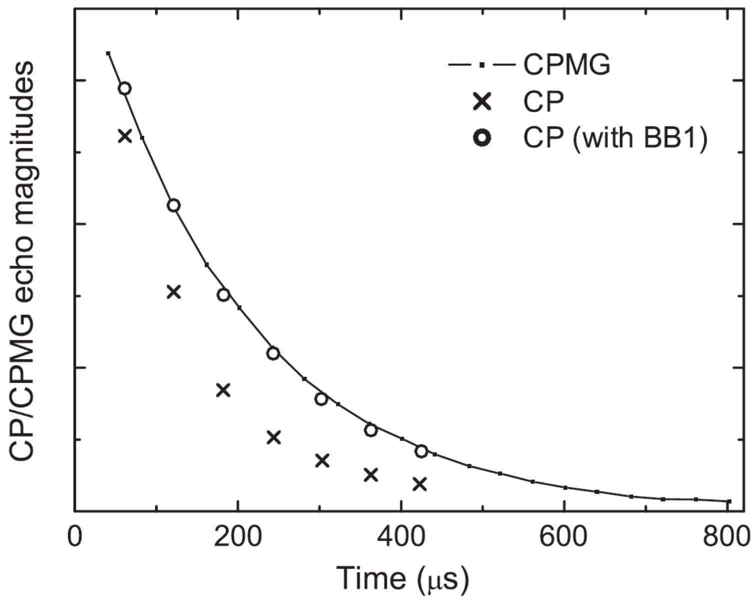
BB1 can be extended to make robust two qubit controlled phase gates



In a system with two different spin–spin couplings the small term distorts evolution under the large term. BB1 controlled phase gates suppress this distortion.

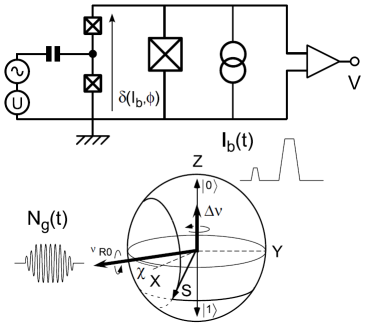
BB1 and electron spins



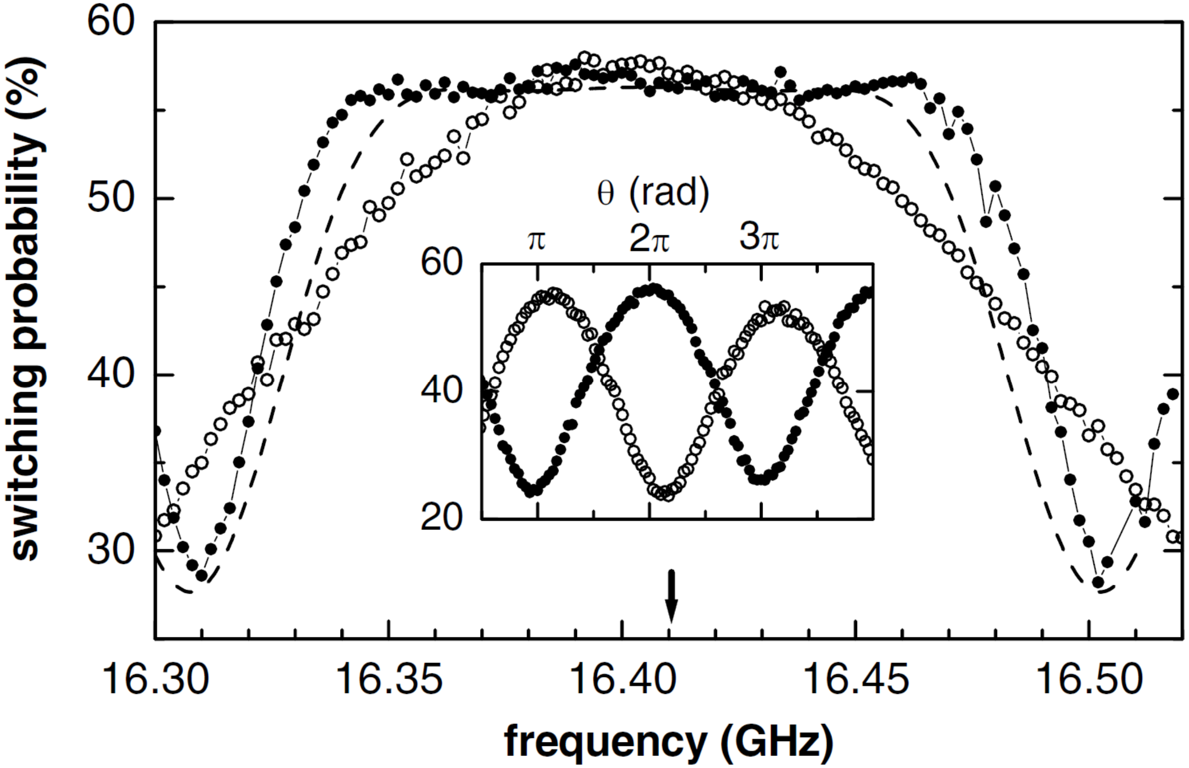


Electron paramagnetic resonance is just like NMR but with electron spins and microwave control fields. BB1 works just the same

SQUIDs and CORPSE

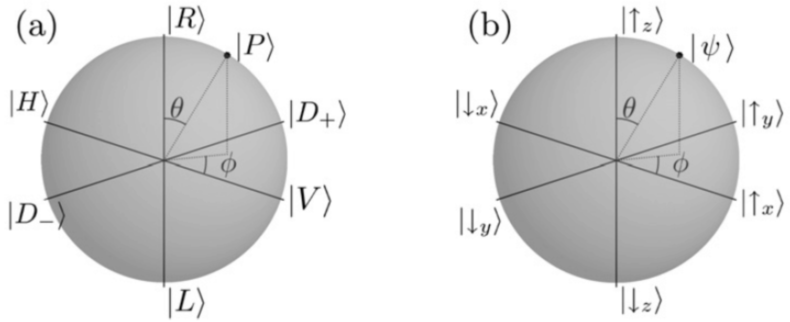


Quantronium SQUID

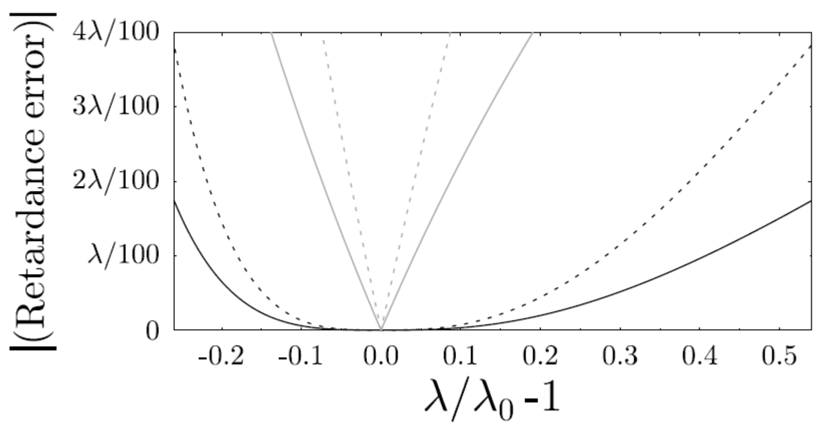


CORPSE gives extremely effective suppression of off-resonance effects

Broadband wave plates



- The Poincaré sphere is equivalent to the Bloch sphere



- Can be used to design wave plates which work well over very large ranges of wavelength
- No experiments yet…

Composite π pulses

- The special case of π pulses is particularly simple, especially for π pulses built from networks of π pulses

- Not particularly useful for quantum logic gates but very useful for *decoupling sequences* which suppress system–bath interactions

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Designing short robust NOT gates for quantum computation

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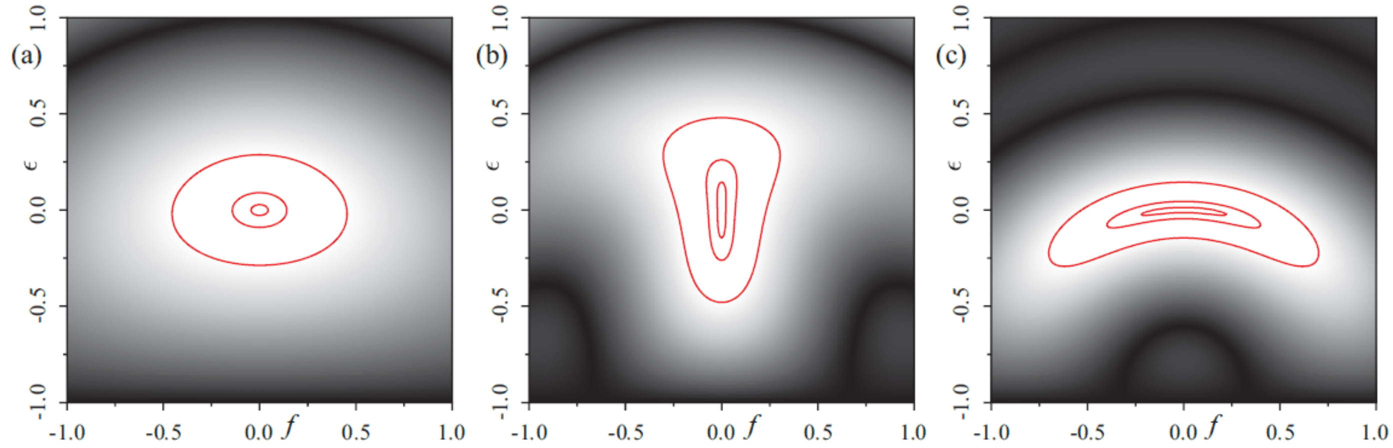
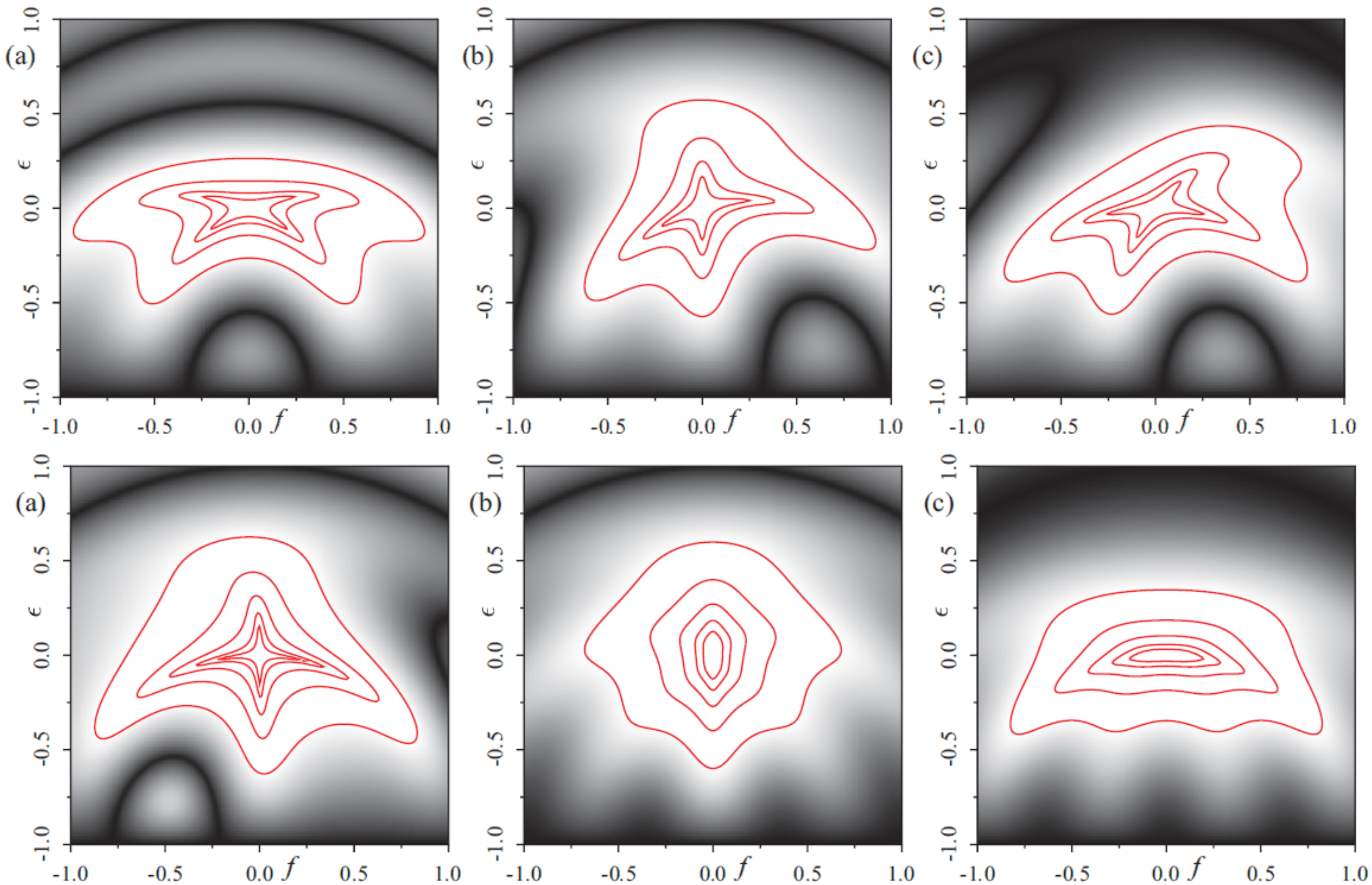


FIG. 2. (Color online) Fidelity achieved by (a) a simple pulse and composite pulses with $n = 3$ optimized to suppress first-order pulse strength errors (b) and off-resonance errors (c). Fidelity is plotted as a function of the fractional pulse strength error ϵ and the off-resonance fraction f. Contours are drawn at 90%, 99%, and 99.9% fidelity, that is, logarithmically spaced infidelities with the inmost contour at an infidelity of 10^{-3}.

With composite rotations made up from 3 pulses it is possible to achieve robustness to either pulse strength error or off resonance error

5

7



With 5 pulses or 7 pulses can do much better and can correct both types of error

Arbitrary precision π pulses

- Can design composite pulses by writing the fidelity as a Taylor series and deleting terms
- F_1 (Wimperis, 1991) removes terms below ε^6
- F_2 removes terms below 18th order
- We hypothesised the F_n family which would remove terms below order 2×3^n
- Now proved: Phys. Lett. A **377**, 2860 (2013)

n	Pulses	Order
0	1	ϵ^2
1	5	ϵ^6
2	25	ϵ^{18}
3	125	ϵ^{54}
4	625	ϵ^{162}

The F_n family of composite π pulses corrects pulse strength errors to arbitrary precision.

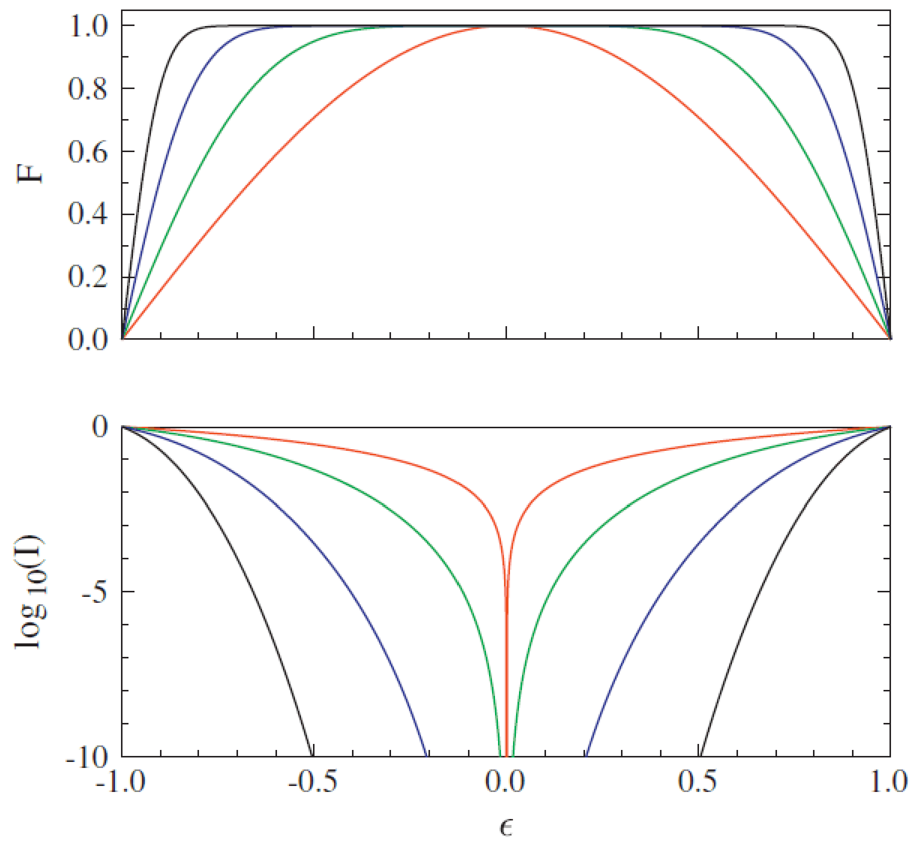
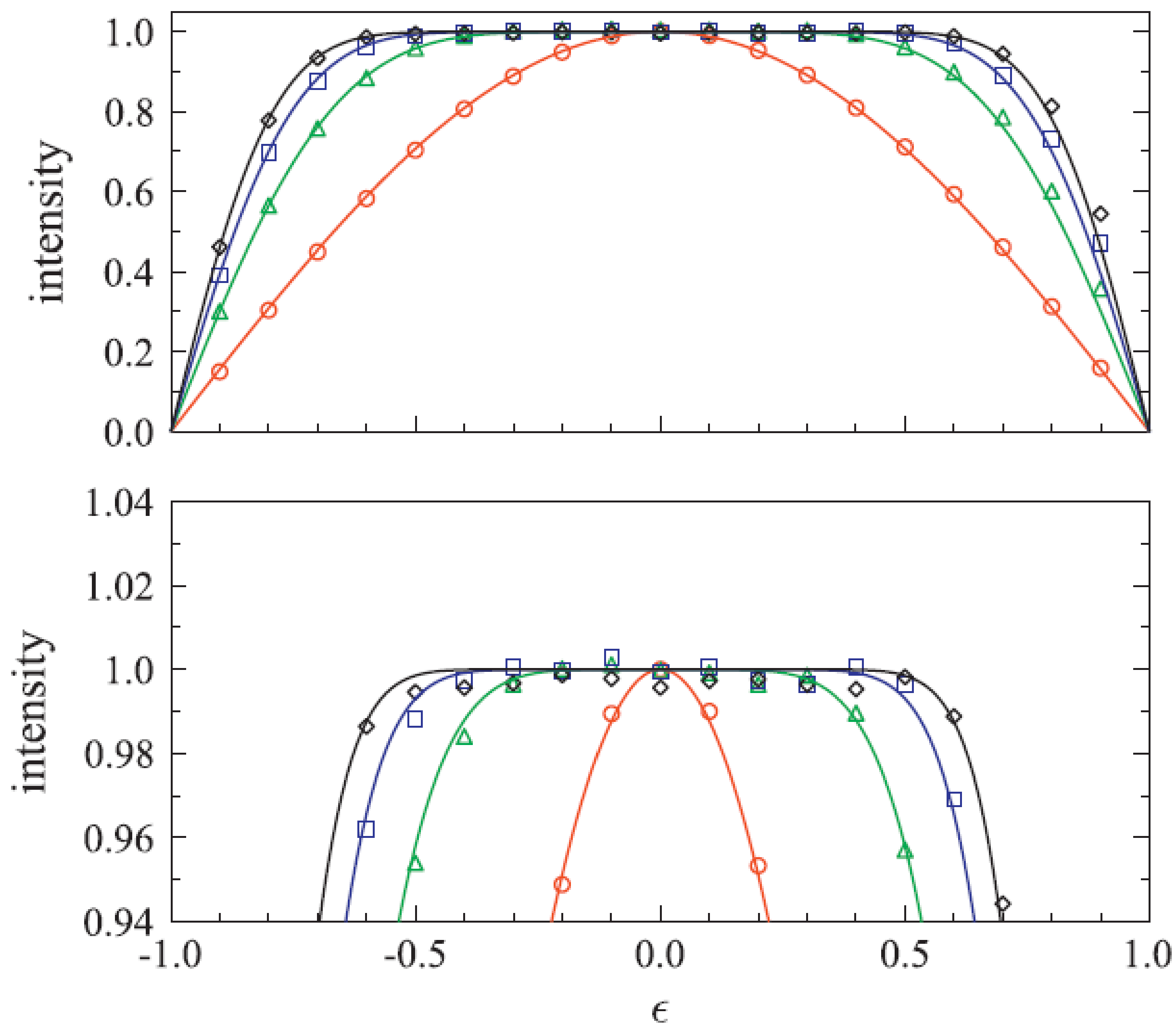


Fig. 1. Fidelity $\mathcal{F}$ and infidelity $\mathcal{I}$ as a function of pulse strength error ϵ for the F_n family of pulses from F_0 (plotted in red) to F_3 (plotted in black). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Arbitrary precision $\pi/2$ pulses

- Taylor series approach works but less well
- Iterative approach of F_n doesn't work
- Numerical searches found the W_n family: a network of 4n π pulses and one $\pi/2$ pulse supressing all errors up to order 4n
- More efficient than F_n but numerical solutions beyond W_4 are *very* hard to find
- Can also make W_n π pulses



Results from NMR experiments with artificial “errors”

Problems

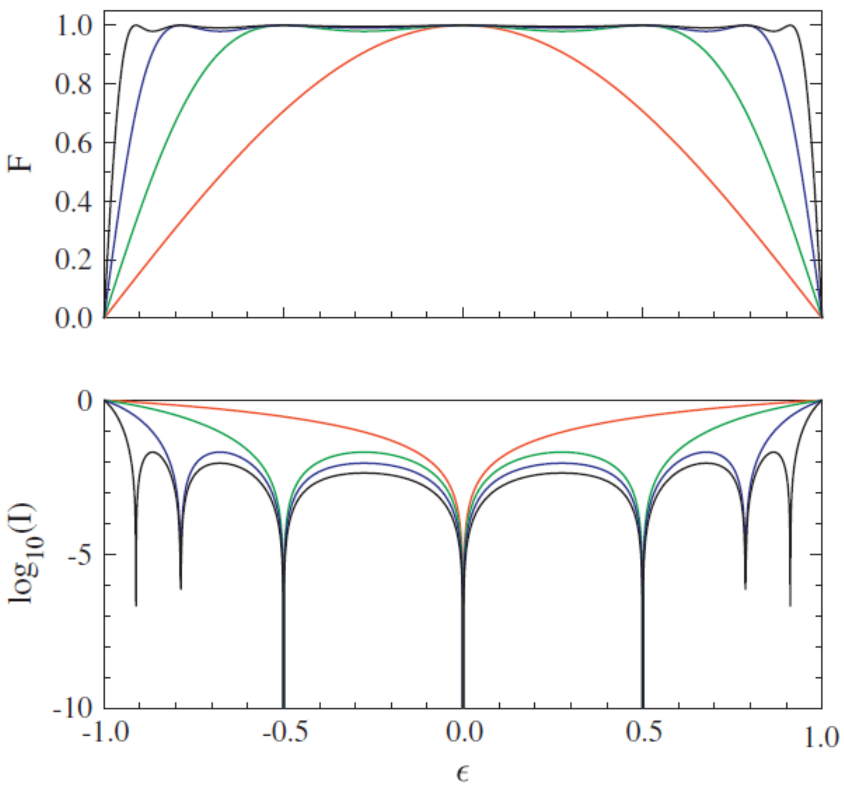
- Numerical solutions beyond W_4 are *very* hard
- Not obvious they even exist

- Guang Hao Low *et al.* have found a way to transform the trigonometrical equations into polynomials, see arXiv:1307.2211
- Can be proved that arbitrary orders exist
- Finding them is still pretty challenging

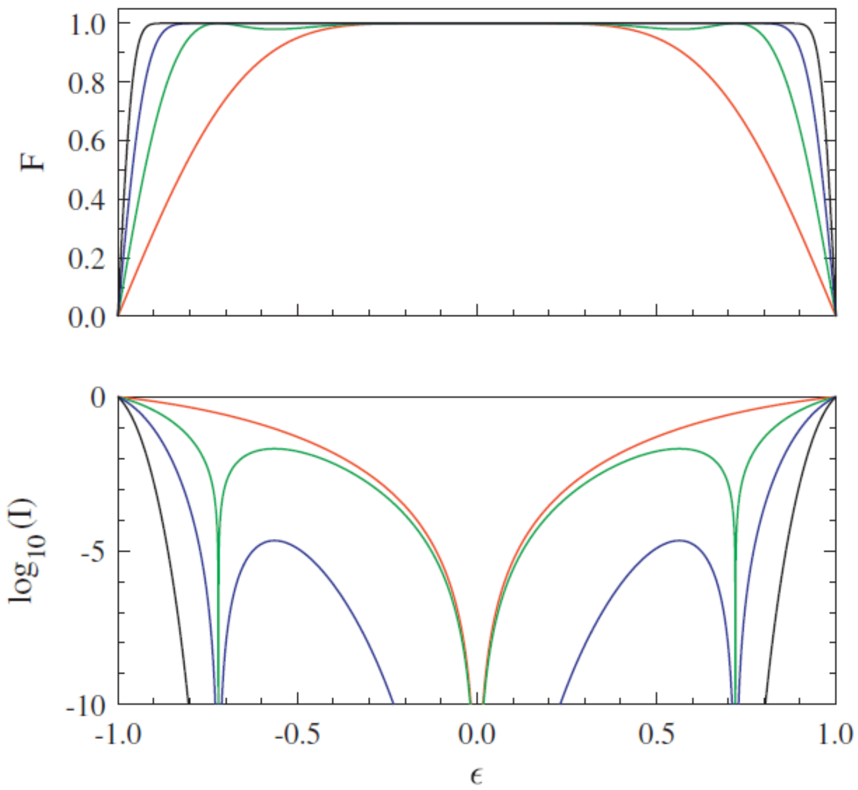
Exotica

- Can find relatives of F_n with exotic reponses
- The G_n family is perfect at certain errors
- The P_n family has passband behaviour (works well for small errors, no effect at large errors)
- The N_n family is hypersensitive to error

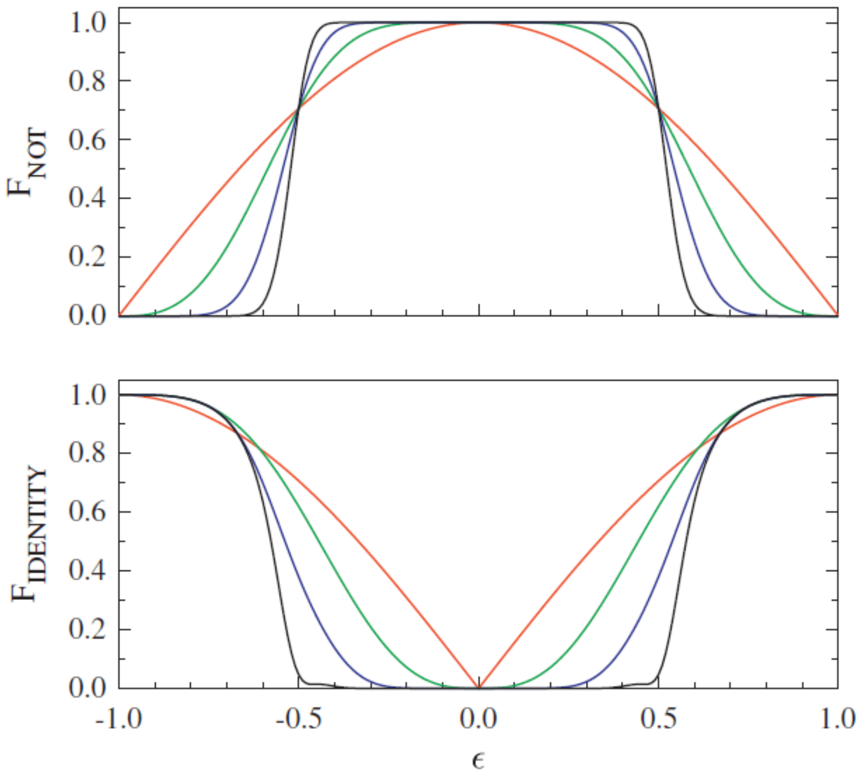
- Families can all be combined in strange ways…



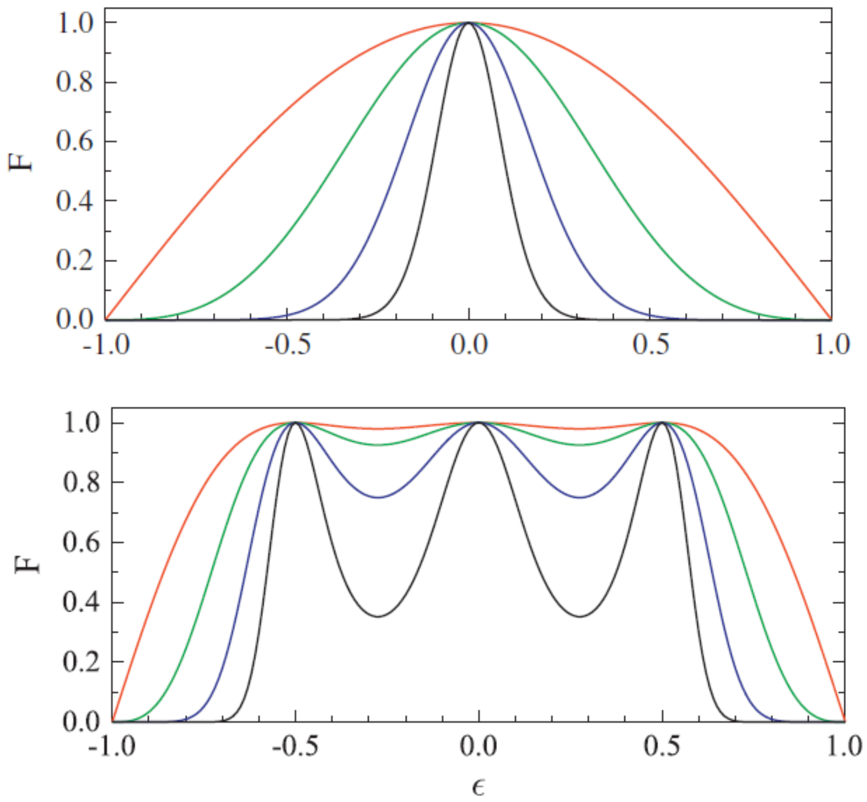
The G_n family from G_0 to G_3



The composite sequences F, GF, FGF and F_2GF



The P_n family from P_0 to P_3



The N_n family and the composite sequences G, NG, N_2G and N_3G

Summary

- Systematic errors are a real problem
- Composite pulses are a good solution in systems with qubit selectivity
- Some pulses work better than others, and theory is not always a good guide
- BB1 is brilliant
- Homonuclear NMR QIP is difficult

The future

- Applying composite pulses more widely
- Still useful work to be done on simultaneous error tolerance
- Still using a lot of trial and error—no good general theory of *why* it works